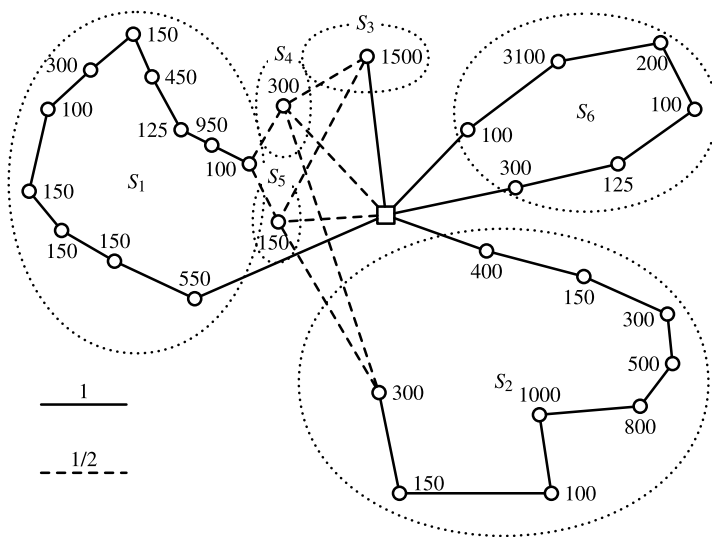


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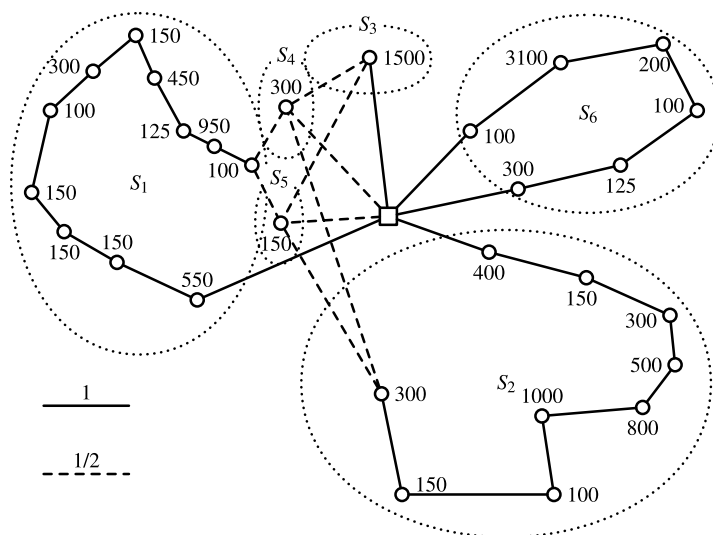
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THE VEHICLE ROUTING PROBLEM



Edited by

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Preface

The Vehicle Routing Problem (VRP) calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers, and it is one of the most important, and studied, combinatorial optimization problems.

More than 40 years have elapsed since Dantzig and Ramser introduced the problem in 1959. They described a real-world application concerning the delivery of gasoline to service stations and proposed the first mathematical programming formulation and algorithmic approach. A few years later, in 1964, Clarke and Wright proposed an effective greedy heuristic that improved on the Dantzig–Ramser approach. Following these two seminal papers, hundreds of models and algorithms were proposed for the optimal and approximate solution of the different versions of the VRP. Dozens of packages for the solution of various real-world VRPs are now available on the market. This interest in VRP is motivated by both its practical relevance and its considerable difficulty: the largest VRP instances that can be consistently solved by the most effective exact algorithms proposed so far contain about 50 customers, whereas larger instances may be solved to optimality only in particular cases.

This book covers the state of the art of both exact and heuristic methods developed in the last decades for the VRP and some of its main variants. Moreover, a considerable part of the book is devoted to the discussion of practical issues.

The realization of this project would have been impossible for us alone to accomplish. We thus involved an enthusiastic group of very well known experts, whose contributions form a large part of the recent history of the VRP (as well as that of Mathematical Programming and Combinatorial Optimization). As editors, we constantly devoted our efforts to reducing as much as possible the overlap between chapters and to preserving coherence and ensuring uniformity of the notation and terminology.

Although focused on a specific family of problems, this book offers a complete overview of the effective use of the most important techniques proposed for the solution of hard combinatorial problems. We, however, assume that readers have a basic knowledge of the main methods for the solution of combinatorial optimization problems (complexity theory, branch-and-bound, branch-and-cut, relaxations, heuristics, metaheuristics, local search, etc.).

The book is divided into three parts, preceded by an introductory chapter in which we present an overview of the VRP family, define the most important variants of the problem, and introduce the main mathematical models. The first part covers the basic and extensively studied version of the VRP, known as capacitated VRP. Three chapters examine the main exact approaches (branch-and-bound, branch-and-cut, and set-covering-based methods), while two other chapters review traditional heuristic approaches and metaheuristics,

respectively. For all methods extensive computational results are analyzed. The second part covers three main variants of the VRP: the VRP with time windows, the VRP with backhauls, and the VRP with pickup and delivery. In each chapter, both exact and heuristic methods are examined. Finally, in the third part, the issues arising in real-world VRP applications, as the presence of dynamic and stochastic components, are discussed by analyzing relevant case studies and presenting software packages.

We warmly thank all the people who contributed to this project, which occupied a considerable amount of the past 3 years: our coauthors, whose competent, patient, and collaborative activity made possible the completion of this volume; the referees whose comments greatly improved the overall presentation; Peter Hammer, editor-in-chief of SIAM Monographs on Discrete Mathematics and Applications, who since the very beginning encouraged us and followed all the steps of the project; and Vickie Kearn, Deborah Poulson, Lou Primus, Sara Triller, Marianne Will, Donna Witzleben, Sam Young, and all the people of SIAM who greatly helped us in the preparation of the overall manuscript.

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